Kleiner-Attractor in a Piecewise-Linear C^1-System

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A maximally simple 3-variable ODE with a single letter-V shaped nonlinearity is shown to reproduce the experimentally discovered Kleiner chaos. Further numerical and analytical studies are encouraged.

Key words: Chaotic Attractors; Spiral-type Chaos; 2-Domain Piecewise-Linear Systems; Dynamical Prototype; Simplicity.

1. Introduction

In 1989, Norman Kleiner discovered a spiral-type chaotic attractor which spirals "the wrong way" (from the outside in) in an electronic circuit [1]. In 1998, a first numerical success in reproducing a similar behavior was reported [2]. In the following, a simpler prototype of the piecewise linear variety is described.

2. The Equation

The proposed system is governed by the equation

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= -0.3x - 0.1y - z, \\ \dot{z} &= 0.1 + x - |x|. \end{aligned} \tag{1}$$

A numerical simulation of (1) is presented in Fig. 1, using the initial conditions given in its legend. A two-dimensional projection of the variables x and y is shown in Figure 2. Other parameter values permit other attractor shapes, including apparently homoclinic Shil'nikov chaos [3] (work in preparation).

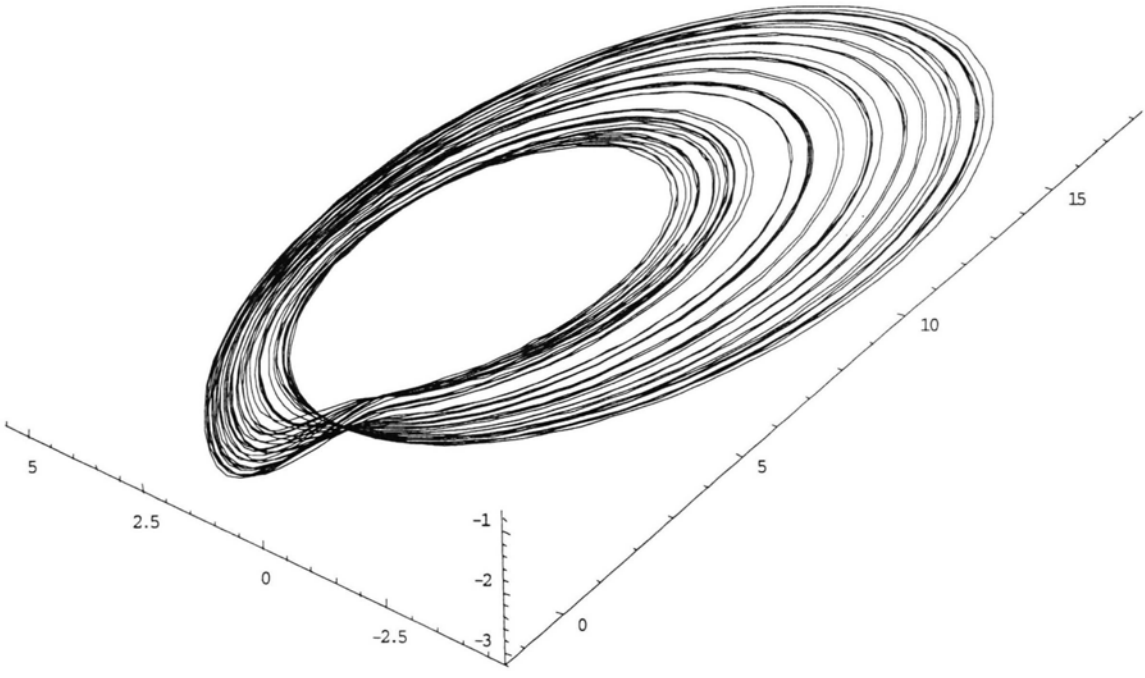

Fig. 1. Kleiner attractor in (1). Numerical simulation using a standard integration algorithm (Runge-Kutta). Initial conditions: $x(0)=6.7$, $y(0)=-2.4$, $z(0)=-1.2$; $t_{end}=600$. The ordinate is the z-axis, the x-axis is to the right, the y-axis to the left.

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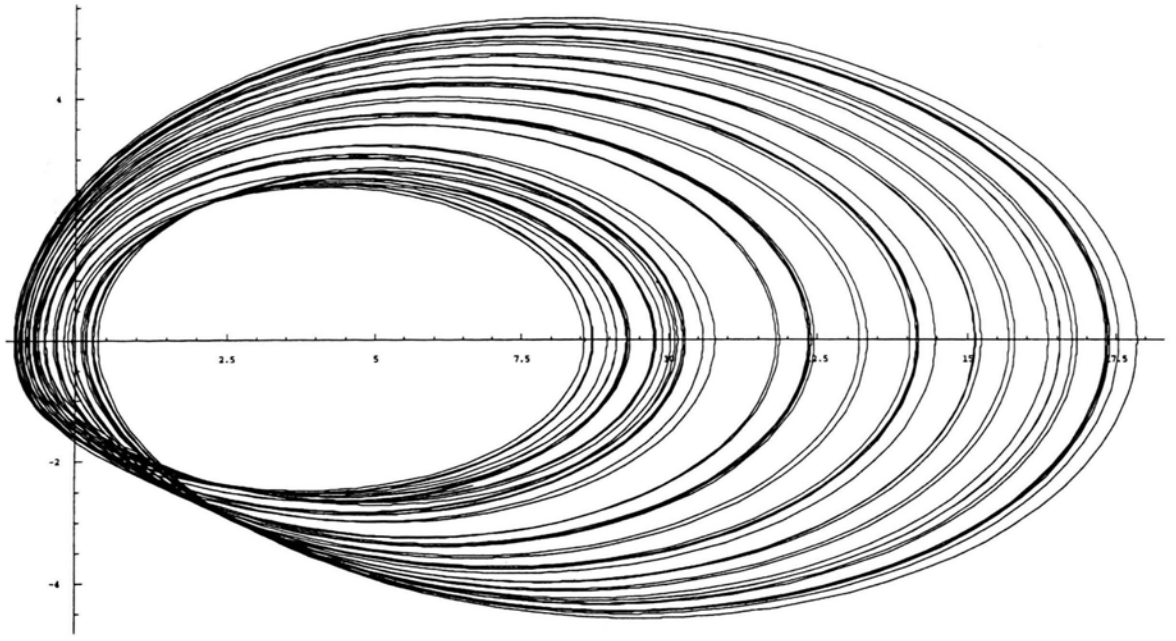

Fig. 2. Two-dimensional projection of Figure 1. The ordinate is the y-axis, the abscissa is the x-axis. The flow is clockwise.

3. Discussion

Surprisingly, a "second zoo" of spiral-type chaotic attractors exists in 3-variable dynamical systems. Hereby the piecewise linear class again functions as a prototype; compare [4] for an example in the first zoo. Analytical studies therefore become an option again (cf. [5, 6]).

Why is Kleiner chaos important? The fact that it was first found in nature and only subsequently explained, shows that it may represent a mature variety of its own in the universe of complex systems.

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